Practical scheme for quantum dense coding between three parties using microwave radiation in trapped ions

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(Dated: February 8, 2007)

We propose a practical scheme for implementing two-dimension quantum dense coding (QDC) between three parties through manipulating three ions confined in microtraps addressed by microwaves and assisted by a magnetic field gradient. The ions in our scheme are not required to be strictly cooled to the vibrational ground state because single-qubit and multi-qubit operations are made via Ising terms, in which the vibrational modes of the ions remain unchanged throughout the scheme, rendering our scheme robust to the heating of the ions. We also present the detailed steps and parameters for implementing the three-party QDC experimentally and show that the proposed scheme is within the current techniques of ion-trap experiments.

PACS numbers: 03.67.-a, 03.67.Lx

Quantum Dense Coding (QDC) [1, 2] is one of the many surprising applications of quantum entanglement in quantum communication. The well-known two-dimension, two-party QDC can transmit two bits of classical information through a quantum channel by actually sending a single qubit. Briefly, if the receiver Alice and the sender Bob previously share a Bell-state entanglement pair and then Bob operates locally on his particle one of the four unitary transformations $\{I, \sigma_x, i\sigma_y, \sigma_z\}$ before sending it to Alice, Alice can obtain two bits of classical information via collective measurement on both particles. Such kind of QDC has been studied extensively for its theoretical properties and security concerns [3, 4], and realized experimentally by using entangled photons [5, 6] and Nuclear Magnetic Resonance(NMR) techniques [7]. Recently, QDC has been a focus again as it was extended to high dimensions and multi parties, which may give rise to many new interesting applications [8]-[14].

Multi-party QDC [11, 12] has not been carried out experimentally till now. Decoherence might be the main obstacle for the implementation of such kind of quantum information processing (QIP) that requires scalability and high fidelity. For example, the trapped ion system, which is known to be a qualified candidate for QIP, is subjected to decoherence that arises mainly from the heating of the trapped ions, and also from imprecise laser and operation instability of optical frequencies [15]. In this paper, we overcome such obstacles for implementing QDC in trapped ion system by using modified ion traps, in which the trapped ions interact via spin-spin coupling produced by a magnetic field gradient and Coulomb force [16, 17, 18]. Single-qubit and multi-qubit operations are thus made via Ising terms, in which the vibrational modes of the ions remain unchanged throughout the scheme. Therefore we do not require the ions to be strictly cooled to the vibrational ground state, rendering our scheme robust to the heating of the ions. In contrast to the original ion trap scheme for QDC [19], we also make use of the microwave source instead of the Raman-type radiation, which makes practical experiments easier than those using lasers. Based on such physical model, we present the concrete steps and suitable parameters for implementing each quantum operation in QDC, and our discussion shows that they are within the current techniques of ion-trap experiments.

I. TWO-DIMENSION, THREE-PARTY QUANTUM DENSE CODING

We'd like to introduce multi-party QDC by focusing on the simplest case of two-dimension QDC between three parties. Assume in the following case that Alice is the only receiver while Bob and Claire are two senders. The three parties previously share a three-qubit GHZ state:

$$|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C) \tag{1}$$

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Then Bob and Claire are to operate locally on their qubits respectively and independently. If their single-qubit unitary operations are chosen appropriately, $|\psi\rangle_{ABC}$ will change into one of the following 8 three-qubit maximal entangled states:

$$\begin{cases} \Phi_{000} = (|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C)/\sqrt{2} \\ \Phi_{001} = (|0\rangle_A|0\rangle_B|1\rangle_C + |1\rangle_A|1\rangle_B|0\rangle_C)/\sqrt{2} \\ \Phi_{010} = (|0\rangle_A|1\rangle_B|0\rangle_C + |1\rangle_A|0\rangle_B|1\rangle_C)/\sqrt{2} \\ \Phi_{011} = (|0\rangle_A|1\rangle_B|1\rangle_C + |1\rangle_A|0\rangle_B|0\rangle_C)/\sqrt{2} \\ \Phi_{100} = (|0\rangle_A|0\rangle_B|0\rangle_C - |1\rangle_A|1\rangle_B|1\rangle_C)/\sqrt{2} \\ \Phi_{101} = (|0\rangle_A|0\rangle_B|1\rangle_C - |1\rangle_A|1\rangle_B|0\rangle_C)/\sqrt{2} \\ \Phi_{110} = (|0\rangle_A|1\rangle_B|0\rangle_C - |1\rangle_A|0\rangle_B|1\rangle_C)/\sqrt{2} \\ \Phi_{111} = (|0\rangle_A|1\rangle_B|1\rangle_C - |1\rangle_A|0\rangle_B|0\rangle_C)/\sqrt{2} \end{cases} \tag{2}$$

Then Bob and Claire should send both their qubits to Alice in quantum channels. Alice then performs a collective measurement to find out which one of the 8 orthogonal states in equation (2) she gets. Since 8 different results can be encoded as a three-bit classical information, Alice achieves, after receiving two qubits and conducting a single measurement, in obtaining three bits of classical information which is partially provided by Bob and partially by Claire. This procedure might have interesting use in quantum communication, *e.g.* if Alice, who has previously let two others help her keep some secret information separately, then wants to retrieve it in a safe way such that neither Bob nor Claire can know it. Furthermore, it is worth mentioning that when extended to arbitrary number of parties, Alice can retrieve N-bit information always by a SINGLE measurement after receiving $N-1$ qubits from $N-1$ senders.

II. OUR PHYSICAL MODEL

In our physical model, the atomic hyperfine levels are encoded as qubits. And due to the magnetic field gradient, the ions can be distinguished in frequency space and individually addressed by microwaves. For pedagogical reasons, our calculations will start from the case of three ions beside each other linearly confined in three individual traps, based on the multi-trap model in Ref.[18]. The three ions, for example Yb^+, are in a magnetic field with gradient in z direction, as shown in figure 1. If we encode qubits as $|F=1, m_F=1\rangle \rightarrow |1\rangle$ and $|F=0\rangle \rightarrow |0\rangle$, the Hamiltonian for this system can be written as:

$$H = \sum_{i=1}^{3} \tfrac{1}{2}\omega_i(z_{0,i})\sigma_{z,i} + \sum_{p=1}^{3} \nu_p a_p^\dagger a_p - \frac{1}{2}J_{13}\sigma_{z,1}\sigma_{z,3} - \frac{1}{2}J(\sigma_{z,1}\sigma_{z,2} + \sigma_{z,2}\sigma_{z,3}) \tag{3}$$

$$J_{ij} = \sum_{p=1}^{3} \frac{\hbar}{2m\nu_p^2} D_{i,p}D_{j,p}\frac{\partial\omega_i(z_{0,i})}{\partial z}\frac{\partial\omega_j(z_{0,j})}{\partial z} \tag{4}$$

where $\omega_i(z_{0,i})$ is the transition frequency for ion i which depends on the equilibrium position $z_{0,i}$ of the ion in the magnetic field gradient, ν_p and $\sigma_{z,i}$ is the p^{th} vibrational frequency and the usual Pauli operator for ion i, and D is the expansion coefficient of the displacement of ion i in terms of the normal mode coordinate. For simplicity, we have adjusted the coupling terms $J_{12} = J_{23} = J$ by simply setting the frequencies of microtrap for ion 1 and 3 to be equal.

Direct algebra calculation shows that the eigenenergies of the equation (3) are as in table I, which can be distinguished from each other. Besides, we give the carrier transition frequency ω_c of a certain ion with corresponding states of the other two ions as in table II. It is obviously that the carrier transition frequencies for each ion are dependent on the states of the other two ions [20].

In our scheme, to resonantly excite one ion irrespective of the states of the other two ions, we should have a microwave with Rabi frequency much larger than the maximum difference between the carrier transition frequencies regarding this ion [16]. Assuming the trap frequencies for microtraps of ion 1, ion 2 and ion 3 are 0.5MHz, 5MHz, and 0.5MHz respectively, the separation of neighboring traps $l = 5\mu m$ and the magnetic field gradient $\partial B/\partial z = 200T/m$, we can estimate $J \approx 10KHz$, much larger than the coupling strength between ion 1 and 3, about 2.0KHz, and the three vibrational mode frequencies are approximately 0.39MHz, 0.64MHz, and 0.72MHz, and the neighboring qubit resonance frequency separation as $g\mu_B\frac{\partial B}{\partial z}l \approx 163MHz$.

Due to the microwave added, we know the interaction Hamiltonian differs from that in the normal linear trap only by replacing the Lamb-Dicke parameter $\eta_{i,p}$ with $\eta'_{i,p} = \sqrt{\eta_{i,p}^2 + \varepsilon_{i,p}^2}$, where the subscript i denotes ion i, p represents

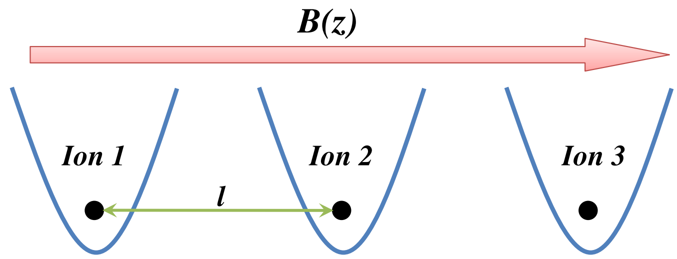

FIG. 1: Schematic plot for multi-trap model: each ion confined in an individual micro-trap, the magnetic field $B(z) = B_0 + \frac{\partial B}{\partial z} z$. l is the separation between neighboring traps.

TABLE I: The eigenenergies in the space spanned by $|0\rangle_1|0\rangle_2|0\rangle_3$, $|1\rangle_1|0\rangle_2|0\rangle_3$, $|0\rangle_1|1\rangle_2|0\rangle_3$, $|0\rangle_1|0\rangle_2|1\rangle_3$, $|1\rangle_1|1\rangle_2|0\rangle_3$, $|1\rangle_1|0\rangle_2|1\rangle_3$, $|0\rangle_1|1\rangle_2|1\rangle_3$, $|1\rangle_1|1\rangle_2|1\rangle_3$.

Eigenvectors (Basis)	Eigenenergies
$\|0\rangle_1\|0\rangle_2\|0\rangle_3$	$-0.5(\omega_1+\omega_2+\omega_3) - J - 0.5J_{13}$
$\|1\rangle_1\|0\rangle_2\|0\rangle_3$	$-0.5(-\omega_1+\omega_2+\omega_3) + 0.5J_{13}$
$\|0\rangle_1\|1\rangle_2\|0\rangle_3$	$-0.5(\omega_1-\omega_2+\omega_3) + J - 0.5J_{13}$
$\|0\rangle_1\|0\rangle_2\|1\rangle_3$	$-0.5(\omega_1+\omega_2-\omega_3) + 0.5J_{13}$
$\|1\rangle_1\|1\rangle_2\|0\rangle_3$	$0.5(\omega_1+\omega_2-\omega_3) + 0.5J_{13}$
$\|1\rangle_1\|0\rangle_2\|1\rangle_3$	$0.5(\omega_1-\omega_2+\omega_3) + J - 0.5J_{13}$
$\|0\rangle_1\|1\rangle_2\|1\rangle_3$	$0.5(-\omega_1+\omega_2+\omega_3) + 0.5J_{13}$
$\|1\rangle_1\|1\rangle_2\|1\rangle_3$	$0.5(\omega_1+\omega_2+\omega_3) - J - 0.5J_{13}$

the p^{th} collective motional mode, $\eta_{i,p}$ is about 0.7×10^{-6}. In the Paschen-Bach limit, the frequency gradients are independent of z, therefore $\varepsilon_{i,p} = D_{i,p}(\sqrt{\hbar/2m\nu_p}\partial B/\partial z)/\nu_p$. With the parameters given in the above example, the maximum is about 0.013, so all the $\eta'_{i,p}$ are much smaller than 1. Direct calculation shows that under the restriction $\varepsilon_{max} < 0.05$ (ε_{max} denotes the maximum of all ε), the smaller the neighboring trap distance l, the bigger the J obtainable. Besides, it can be easily shown that the evolution operator for addressing ion i by a microwave in the interaction picture is,

$$U_I^i(\theta,\phi) = \begin{pmatrix} \cos\frac{\theta}{2} & -ie^{-i\phi}\sin\frac{\theta}{2} \\ -ie^{+i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \tag{5}$$

where $\theta = \Omega t$ with the Rabi frequency Ω being in the order of MHz, and phase ϕ relating to the position of ion in the microwave. Equation (5) achieves the single qubit-rotation operation that is needed in the implementation of QDC protocol.

To further attain the two-qubit controlled operation, we borrow the technique of realizing Controlled-NOT gate from NMR quantum computing [21], where the CNOT operation is achieved through a sequence of pulses. In product operator representation, that is:

$$\begin{aligned} U_{CNOT}(i,j) = {} & e^{-i\pi/4} e^{-i\pi/4\sigma_{y,j}} e^{i\pi/4\sigma_{z,i}} e^{i\pi/4\sigma_{z,j}} \\ & \times e^{-i\pi/4\sigma_{z,i}\sigma_{z,j}} e^{i\pi/4\sigma_{y,j}} \end{aligned} \tag{6}$$

where ion i and j act as the controlling qubit and target qubit respectively. $e^{-i\pi/4\sigma_{z,i}\sigma_{z,j}}$ can be realized through the

TABLE II: The carrier transition frequency ω_c of a certain ion with corresponding states of the other two ions.

State for ion 2,3	ω_c of ion 1	State for ion 1,3	ω_c of ion 2	State for ion 1,2	ω_c of ion 3
$\|1\rangle\|1\rangle$	$\omega_1 - J - J_{13}$	$\|1\rangle\|1\rangle$	$\omega_2 - 2J$	$\|1\rangle\|1\rangle$	$\omega_3 - J - J_{13}$
$\|1\rangle\|0\rangle$	$\omega_1 - J + J_{13}$	$\|1\rangle\|0\rangle$	ω_2	$\|1\rangle\|0\rangle$	$\omega_3 + J - J_{13}$
$\|0\rangle\|1\rangle$	$\omega_1 + J - J_{13}$	$\|0\rangle\|1\rangle$	ω_2	$\|0\rangle\|1\rangle$	$\omega_3 - J + J_{13}$
$\|0\rangle\|0\rangle$	$\omega_1 + J + J_{13}$	$\|0\rangle\|0\rangle$	$\omega_2 + 2J$	$\|0\rangle\|0\rangle$	$\omega_3 + J + J_{13}$

coupling term $\frac{1}{2}J_{i,j}\sigma_{z,i}\sigma_{z,j}$ in equation (3). The undesired evolution brought by the other terms in equation (3) can be eliminated by the refocusing techniques [2]. All the other terms can be implemented by microwave pulses.

In table III we give a concrete example of $U_{CNOT}(1,2)$. Note that in the context of NMR, Rabi frequency is much larger than other characteristic frequencies. In this case, $\omega_i \sim 13GHz$ is the biggest frequency, much larger than other characteristic frequencies. Therefore, to avoid undesired evolution due to $\omega_i(z_{0,i})$, we have to carefully control the pulse length t to satisfy $\omega_i(z_{0,i})t = 2n\pi$, $(n = 1, 2, 3, \ldots)$, which can be realized by properly adjusting B_0, $\partial B/\partial z$ and Ω. Given the parameters in table III for $l = 5\mu m$, the total time for completing the CNOT gate is about 2.82ms. Similar parameters can be chosen to realize $U_{CNOT}(2,3)$ too.

TABLE III: The pulse sequences and estimated time for implementing each term in $U_{CNOT}(1,2)$, with U_0 being the unitary evolution operator of the Hamiltonian in equation (3) without the second term, and t_0 being the implementation time for U_0. Assume the implementation time for any single qubit rotation $U_I^i(\theta,\phi)$ to be $5\mu s$. J is at the order of kHz, which could be achieved by properly adjusting the Rabi frequency of the microwave.

Term	Pulse sequences	Estimated time
$e^{i\pi/4\sigma_{y,1}}$	$U_I^1(\frac{\pi}{2},\frac{\pi}{2})$	$5\mu s$
$e^{-i\pi/4\sigma_{z,1}\sigma_{z,2}}$	$U_I^1(\pi,0)U_I^2(\pi,0)U_0(\frac{t_0}{4})$ $U_I^3(\pi,0)U_0(\frac{t_0}{4})U_I^1(\pi,0)$ $U_I^2(\pi,0)U_0(\frac{t_0}{4})U_I^3(\pi,0)$ $U_0(\frac{t_0}{4})(t_0 = \frac{7\pi}{2J})$	$2.78ms$
$e^{i\pi/4\sigma_{z,1}}$	$U_I^1(\frac{\pi}{2},\frac{\pi}{2})U_I^1(\frac{\pi}{2},0)U_I^1(\frac{7\pi}{2},\frac{\pi}{2})$	$15\mu s$
$e^{i\pi/4\sigma_{z,2}}$	$U_I^2(\frac{\pi}{2},\frac{\pi}{2})U_I^2(\frac{\pi}{2},0)U_I^2(\frac{7\pi}{2},\frac{\pi}{2})$	$15\mu s$
$e^{-i\pi/4\sigma_{y,1}}$	$U_I^1(\frac{7\pi}{2},\frac{\pi}{2})$	$5\mu s$

III. THE IMPLEMENTATION

Suppose the qubits hold by Alice, Bob and Claire are represented by ion 1, 2, 3 respectively, which are initially in the state $|0\rangle_1|0\rangle_2|0\rangle_3$. Then we first need to prepare the three-qubit GHZ state in equation (1), which can be attained by first applying a Hadamard operation on ion 1, followed by a CNOT operation on ion 1, 2, and finally a CNOT operation on ion 2, 3:

$$|0\rangle_1|0\rangle_2|0\rangle_3 \xrightarrow{H(1)} \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)|0\rangle_2|0\rangle_3 \tag{7}$$

$$\xrightarrow{U_{CNOT}(1,2)} \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|0\rangle_3 \tag{8}$$

$$\xrightarrow{U_{CNOT}(2,3)} \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1|1\rangle_2|1\rangle_3) \tag{9}$$

Next comes the problem of which set of specific single-qubit unitary operations in equation (5) Bob and Claire should apply to their qubits. We generally prefer to use the set of Pauli operators $\{I, \sigma_x, i\sigma_y, \sigma_z\}$, but if Bob and Claire can both choose each of the four Pauli operators, there will be $4 \times 4 = 16$ different results, not in accordance with the 8 orthogonal resultant states in equation (1). In order to prevent confusion in the information Alice finally obtained, here we prescribe that Bob can perform each of $\{I, \sigma_x, i\sigma_y, \sigma_z\}$ but Claire can only perform I and σ_x, such as in Ref. [11]. Note that there are also many other practicable choices, especially when extended to arbitrary number of parties [12]. For three-party case, the choice can only be asymmetric, since Bob actually provides two bits of classical information while Claire provides only one bit. We show in table IV how to use our physical model to implement those single-qubit unitary operations so as to obtain each of the states in equation (1).

The final step is to collectively measure all of the three ions to find out the encoded information. The 8 maximal entangled basis in equation (1) are not easy to distinguish experimentally, but we can apply the following operations to change them into computational basis:

$$|\Phi_{abc}\rangle \xrightarrow{U_{CNOT}(2,3),U_{CNOT}(1,2),H(1)} |abc\rangle \tag{10}$$

where $a, b, c \in \{0, 1\}$ denote the encoded three-bit classical information.

Figure 2 clearly depicts the whole procedure of our scheme in circuit model. Alice conducts a single measurement at the output of the circuit and obtains the encoded information. Till now, we have already shown how to implement all the operations needed in the our whole procedure.

TABLE IV: The state obtained after possible operations through pulse sequences by Bob and Claire. We adopt the same parameters as in table I.

State obtained	Operations by Bob and Claire	Pulse Sequences	Estimated time
Φ_{000}	I, I	No Pulses	0
Φ_{001}	I, σ_x	$U_I^3(\pi, 0)$	$5\mu s$
Φ_{010}	σ_x, I	$U_I^2(\pi, 0)$	$5\mu s$
Φ_{011}	σ_x, σ_x	$U_I^3(\pi, 0), U_I^2(\pi, 0)$	$10\mu s$
Φ_{100}	σ_z, I	$U_I^2(\pi, \frac{\pi}{2})U_I^2(\pi, 0)$	$10\mu s$
Φ_{101}	σ_z, σ_x	$U_I^2(\pi, \frac{\pi}{2})U_I^2(\pi, 0), U_I^3(\pi, 0)$	$15\mu s$
Φ_{110}	$i\sigma_y, I$	$U_I^2(\pi, \frac{\pi}{2})$	$5\mu s$
Φ_{111}	$i\sigma_y, \sigma_x$	$U_I^2(\pi, \frac{\pi}{2}), U_I^3(\pi, 0)$	$10\mu s$

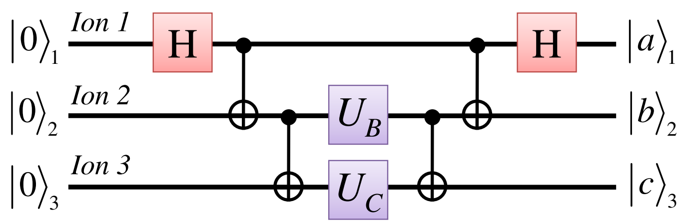

FIG. 2: Circuit model of the our scheme for implementing the two-dimension QDC between three parties. H denotes the Hadamard gate, U_B and U_C stand for the single-qubit unitary operations Bob and Claire perform on the qubits at their hands.

IV. DISCUSSIONS

It is necessary to give some brief discussions on the experimental feasibility of our scheme. Compared with the Cirac-Zoller gate operation [22] and the geometric phase gate operation [19, 23], our scheme does not require moving or hiding any ions, which is important in view of decoherence [24]. Based on the table III, we estimate the two-qubit CNOT gate operation time to be 3 ms. As an example, we supposed 10% of the modes are really excited during our implementation of the gate, then our scheme would work as long as the heating times of the vibrational states are longer than 0.3 ms. which meets the requirement of current technology. Besides, the microtrap in our consideration is of the size of $5\mu m$, and experimentally, three ions in a linear trap with spacing of the order of μm has already been achieved [22]. In addition, microwave with a certain bandwidth and a rapid change of phase is a perfect technique and magnetic field gradient up to 8000T/m is within the reach of current experiments [25]. In-trap magnetic field gradient of 200 T/m over several micrometers, which is used in our scheme, can be easily achieved through methods similar to those presented in Ref. [26, 27].

Finally, as previously shown by Wunderlich [17], the magnetic field gradient allows the qubits to be individually addressed in the microwave via Zeeman splitting of the hyperfine structure, and the qubit readout process can be achieved through trapped ion fluorescence shelving techniques. We know that the fluorescence detecting measurement takes $250\mu s$ [22] base on the parameters in table III, IV, thus we can estimate that the whole quantum dense coding process takes less than 6ms. The implementation time can be further shortened by increasing the spin-spin coupling strength J, which can be done by reducing the inter-trap spacing l or enlarging the magnetic field gradient.

Our scheme is implemented in linear ion traps, whereas it will also be practicable in multi-trap devices considering currently available techniques [22]. Thus our scheme can in principle be extended to QDC between arbitrary number of parties, as theoretically proposed in Ref. [11, 12], where the maximal entangled state can still be prepared through Hadamard and CNOT gates, and the Pauli operations each sender performs can be carried out using the same evolution operator as in equation (5). It is our hope that our scheme can guide future experiment for these QDC procedures, as well as stimulating further discourse on related QIP techniques in real physical systems.

Acknowledgments

The authors would thank Prof. Ying Wu for many enlightening discussions and helpful suggestions. This work was partially supported by National Fundamental Research Program of China 2005CB724508 and by National Natural

Science Foundation of China under Grant Nos. 60478029, 90503010, 10634060 and 10575040.

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